

Научном већу Математичког института САНУ

Директору Математичког института САНУ

Руководиоцу пројекта ОИ 174001

Извештај са научног скупа „9th European Nonlinear Dynamics Conference" (ENOC 2017) у Будимпешти, организованог под покровитељством „European Mechanics Society“ (EUROMECH)

Поштовани,

од 25. до 30. јуна 2017. године одржана је конференција под називом „9th European Nonlinear Dynamics Conference" (ENOC 2017) у Будимпешти, Мађарска. На конференцији сам учествовао са усменим саопштењем на минисимпозијуму „Nonlinear Dynamics in Engineering Systems“ којим су председавали проф. др Камуца (Стевановић) Хедрих и проф. др Yuri Vladimirovich Mikhlin са радом под називом „Dynamics of ball bearings with damages at outer raceway surface - vibration response under different loads“, и са постером са истим насловом у постер секцији. Аутори рада су Ивана Атанасовска и Наташа Солдат које су ми предложиле да представим резултате њиховог истраживања, што сам прихватио и успешно урадио. Моје излагање је наишло на позитиван одазив и било ми је постављено неколико питања.

На конференцији је учествовао велики број истраживача са иностраних универзитета али и осам истраживача из Србије од чега пет истраживача са пројекта ОИ174001 укључујући и руководиоца пројекта. Конференција је организована у оквиру 21 минисимпозијума. Области које је покривала ова конференција су: моделирање и методе у нелинеарној динамици, примена фракционог рачуна на проблеме механике, квалитативна и квантитативна анализа нелинеарних динамичких система, нелинеарна динамика континуалних, дисконтинуалних и хибридних система, бифуркација и хаос, нелинеарни стохастички системи, нелинеарни динамички феномени, проблеми управљања осцилацијама и хаосом, примена механике на више скала и у реалним инжењерским проблемима из области грађевинарства, електронике, механике, медицине, материјала, комуникација и др. Такође, конференција је обухватала и мултидисциплинарне теме из примењене математике, физике, биофизике, генетике, нанотехнологија, финансија, медицине и гео-наука.

Након мог излагања рада „*Dynamics of ball bearings with damages at outer raceway surface - vibration response under different loads*“, било је великог интересовања из публике и пуно питања, нарочито истраживачке групе са Департмана за механику, Универзитета Tianjin из Кине, који се баве истом научном облашћу. Дошло је до договора за научну сарадњу у будућности у смислу научних публикација и билателарних пројеката. Проф. др Катица (Стевановић) Хедрих је позвала колеге са Универзитета Tianjin да пошаљу радове у српске научне часописе за механику и математику и да посете Математички институт САНУ и одрже предавање на семинару за механику, а они су изразили жељу да то и ураде.

Радни део конференције је одржан у згради Универзитета Технолошких и Економских наука у Будимпешти. Конференција се одржава на три године.

За учешће на овој конференцији сам подржан од Математичког института САНУ, пројекта "ОИ174001 Динамика хибридних система сложених структура, Механика материјала" и руководиоца пројекта проф. др Катице (Стевановић) Хедрих и Министарства просвете, науке и технолошког развоја.



дана 03.07.2017.

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Никола Нешић  
истраживач-сарадник, Математички институт САНУ





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- 09.10 ID 354**  
**Particle filters with nudging in multiscale chaotic systems: with application to the Lorenz-96 atmospheric model**  
 Hoong Yeong, Ryne Beeson, Navaratnam Sri Namachchivaya  
*University of Illinois at Urbana-Champaign, Aerospace Engineering, Urbana, USA*
- 09.30 ID 457**  
**Experimental identification of an aircraft piccolo tube exhibiting nonsmooth nonlinearities**  
 Tilan Dossogne<sup>1</sup>, Maarten Schoukens<sup>2</sup>, Bruno Bernay<sup>3</sup>, Jean-Philippe Noel<sup>1</sup>, Gaetan Kerschen<sup>1</sup>  
<sup>1</sup>*University of Liege, Aerospace and Mechanical Engineering, Liege, Belgium*  
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<sup>3</sup>*SONACA SA, Icing and Dynamic Simulation, Gosselies, Belgium*
- 09.50 ID 490**  
**Model reduction for mercury porosimetry: invasion percolation on regular, exotic and random networks**  
 Bendegúz Dezső Bak  
*Budapest University of Technology and Economics, Department of Fluid Mechanics, Budapest, Hungary*

**10.30 - 11.00 Coffee break**

**Room 1 (KF51)**

- 11.00 - 12.00 Keynote lecture**
- Autonomous assembly of a team of flexible spacecraft**  
 Haiyan Hu  
*School of Aerospace Engineering, Beijing Institute of Technology, Beijing, China*

**12.00 - 13.30 Lunch break**

**Room 1 (KF51)**

- 13.30 - 15.30 MS 09 / IV.**  
**Nonlinear Dynamics in Engineering Systems**
- Chair:** Yuri Vladimirovich Mikhlin      **Co-chair:** Katica Hedrih

13.30

**ID 158**

**Dynamics of ball bearings with damages at outer raceway surface - vibration response under different loads**

Ivana Atanasovska<sup>1</sup>, Natasa Soldat<sup>2</sup>

*<sup>1</sup>Mathematical Institute of Serbian Academy of Sciences and Arts,  
Department of Mechanics, Belgrade, Serbia*

*<sup>2</sup>University of Belgrade - Faculty of Mechanical Engineering,  
Machine Design Department, Belgrade, Serbia*

13.50

**ID 265**

**Nonlinear rotordynamic-thermal analysis of micro gas turbines**

Frans Duijnhouwer, Rob Fey, Henk Nijmeijer

*Eindhoven University of Technology, Department of Mechanical Engineering,  
Eindhoven, The Netherlands*

14.10

**ID 281**

**Torsional vibrations in truck powertrains with dual mass flywheel having piecewise linear stiffness**

Lina Wramner

*Chalmers University of Technology, Applied Mechanics, Gothenburg, Sweden*

14.30

**ID 308**

**Non-linear dynamics of a rotor system with compliant seal**

Simon Baeuerle, H. Hetzler

*University of Kassel, Engineering Dynamics Group, Kassel, Germany*

14.50

**ID 342**

**Non-linear dynamics of a heavy mass particle and rolling ball along curvilinear trace of series of circle arcs: Phase trajectory portraits, some analogies and vibro-impacts**

Katica Hedrih (Stevanovic)

*Mathematical Institute of Serbian Academy of Sciences and Arts,  
Department of Mechanics, Belgrade, Serbia*

15.10

**ID 22**

**Evaluating nonlinear responses of asphalt concrete mixtures under time-dependent loading: in view of three representation functions**

Chun-Hsing Ho, Cristina Pilar Martin Linares

*Northern Arizona University, Department of Civil and Environmental Engineering,  
Flagstaff, USA*

TUESDAY

# Dynamics of Ball Bearings with Damages at Outer Raceway Surface -Vibration Response under Different Loads

Ivana Atanasovska\* and Natasa Soldat\*\*

\* *Mathematical institute of SASA, Belgrade, Serbia*

\*\* *University of Belgrade-Faculty of Mechanical Engineering, Serbia*

**Summary.** This paper presents the new approach in analyzing the vibration response of rolling ball bearings. The simplified methodology for vibration calculation is developed in order to become a tool for analyzing the wide range of different ball bearing types with different defects and under various working conditions. The methodology is verified by compare with available experimental results. For a particular ball bearing type and a particular damage shape and size, all of steps in the developed procedure are performed and results are presented. The conclusions about the dynamics of ball bearing with damage are obtained in accordance with graphical presentation of the results.

## Introduction

The operation characteristics which are required from rolling bearings become very rigorous in last few decades. This condition is result of the fact that rolling bearings are the part of almost all contemporary transmission systems including systems in generators of renewable energy. Consequently, the energy savings, longer working life and noise and vibration reducing have become an integral part of all ball bearing studies. In that sense, the rolling bearings as well know and widely used standard machine elements, have been entered into the focus of contemporary research of many group of mechanics scientists. Almost of all them are studying the dynamic behavior of rolling bearings as the most important bearing characteristics which reflect almost all characteristics of design, production, assembly and bearing operation and maintenance.

But, the unique methodology for analyzing the dynamic behavior of rolling bearings still doesn't exist. The results presents in this paper is a step forward in developing such a methodology, which will be in same time with satisfactory accuracy and simplified enough for widely usage, with emphasis on qualitative and comparative analyses. The special attention is paid to developed procedure with possibilities to studying the different ball bearing defects with failure prediction possibilities, in order to make a tool for reversible maintenance.

## Simplified methodology for analyzing the dynamics of radial ball bearings

Ball bearing assembly has been analyzed as a system of elastic related masses, with fixed outer ring in housing, and rigidly bounded inner ring to the shaft. Contact between balls and races can be modeled as non-linear springs, which operate only in compression, simulating contact deformations and resulting forces. System of differential equations describing the vibrations of the bearing assembly can be expressed as, [1]:

$$[M]\{\ddot{q}(t)\} + [D]\{\dot{q}(t)\} + [C]\{q(t)\} = \{F(t)\} \quad (1)$$

where:  $[M]$  – mass matrix;  $[D]$  – damping matrix;  $[C]$  – stiffness matrix;  $\{q(t)\}$  and  $\{F(t)\}$  are vectors of generalized movements and external forces.

The main postulates of the mathematical phenomenological mapping has been used for simplifying the system of equations (1) in the case of radial ball bearing, [1], to a single degree of freedom system with reduced mass of shaft and housing and with time-dependence radial stiffness of ball bearing of whole assembly.

The time-dependence stiffness has been calculated by Finite Element Analysis, [2]. In figure 1 the developed Finite Element Model of a particular radial ball bearing type 6206 with defect (with width of 0,3mm and depth of 50 $\mu$ m) is shown. The modeled defect shape often exists under fatigue or other damaging phenomena, [3, 4]. Figure 2 shows obtained radial stiffness for different external loads.

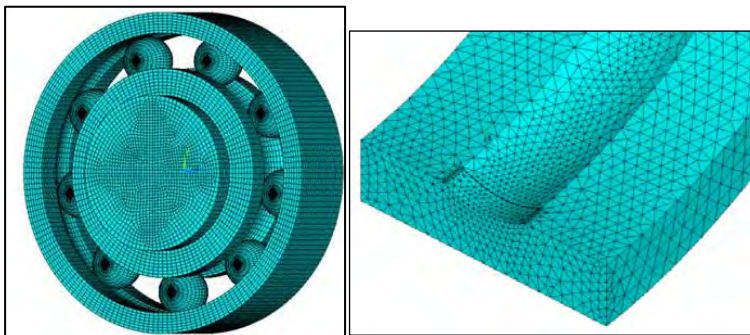


Figure 1. Finite Element Model for stress-strain analysis (with damage detail)

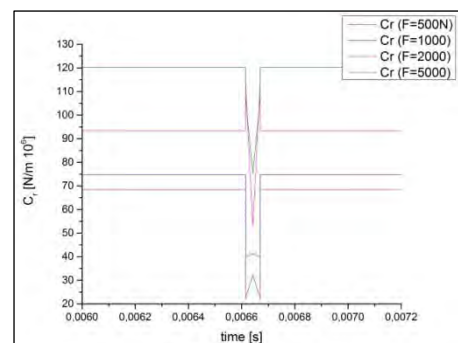


Figure 2. Variable radial stiffness for ball bearing with damage - zoom in detail



## Results and conclusions

The obtained functions of radial stiffness are incorporated in the appropriate equations for calculation the radial ball bearing vibrations, [1]. The Runge-Kutta method by MathLab software has been used for differential equations solving. The obtained results give the vibration response of modeled ball bearing type in both of the cases: with and without damages and for different external loads. For new methodology verification the available experimental results for the same type of ball bearing vibrations in the case without damages are used, [5].

The obtained results are presented by set of comparative diagrams. Two of the comparative diagrams for discussing the influence of damages on 6206 bearings vibrations are shown in Fig. 3. Similar diagrams are created for other external forces in range of ball bearing load capacity. It is easy to conclude that the effects of damages on other raceway surface have different character for different external loads. The no predictable qualitative vibration response of rolling bearings with damages could be expected. Therefore, the one more quality of presented methodology could be the analysis of optimal external load range in cases of rolling bearings for which operation the monitoring systems shows the responses characteristic for existing damages. For these purposes the velocity-displacement diagrams, Fig.4, could be created and analyzed.

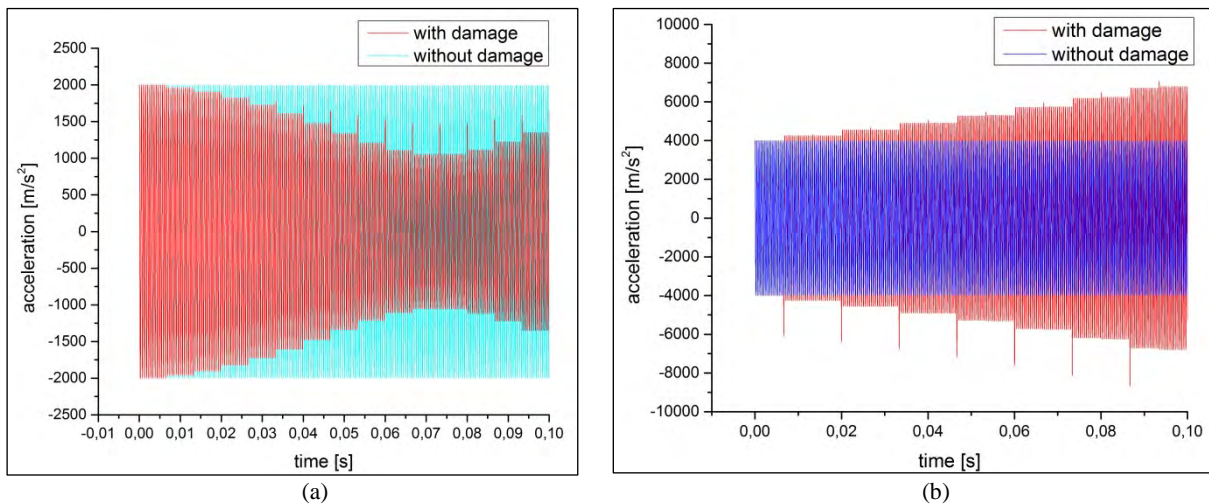


Figure 3. Comparative diagrams for vibration responses of ball bearing with and without damage: (a) external load 1000 N; (b) external load 2000 N

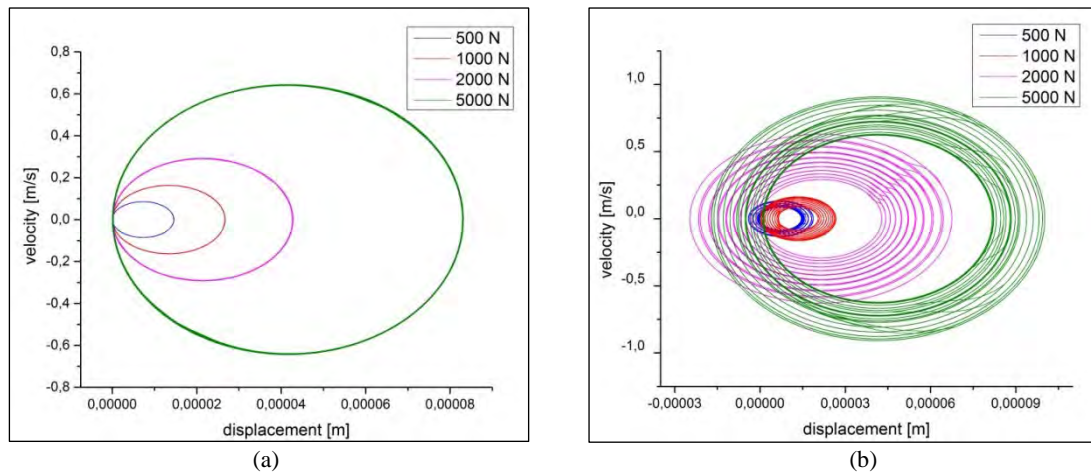
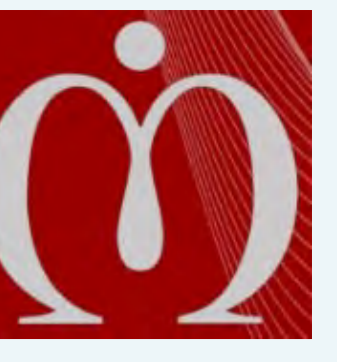


Figure 4. Velocity-displacement diagrams: (a) ball bearing without damage; (b) ball bearing with damage

## References

- [1] Atanasovska I. (2015) The Mathematical Phenomenological Mapping in Nonlinear Dynamics of Spur Gear Pair and Radial Ball Bearing due to the Variable Stiffness. *International Journal of Non-linear Mechanics* **73**:114-120.
- [2] Atanasovska I. et al. (2015) Developing the finite element model for dynamic analysis of radial ball bearing. *5<sup>th</sup> International Congress of Serbian Society of Mechanics, Serbia*: 1-6.
- [3] Ding W., Zhang Z., Zha F. (2015) Vibration response of ball bearings with different defect sizes in the outer raceway: Simulation with a 3-D finite element model. *The 14th IFToMM World Congress, Taipei, Taiwan*, DOI: 10.6567/IFToMM.14TH.WC.OS14.005.
- [4] Cui L. et al. (2016) Vibration response mechanism of faulty outer race rolling element bearings for quantitative analysis. *Journal of Sound and Vibration* **364**: 67-76.
- [5] Tomovic R. et al. (2010) Vibration response of rigid rotor in unloaded rolling element bearing. *International Journal of Mechanical Sciences* **52**: 1176-1185.



# Dynamics of Ball Bearings with Damages at Outer Raceway Surface -Vibration Response under Different Loads

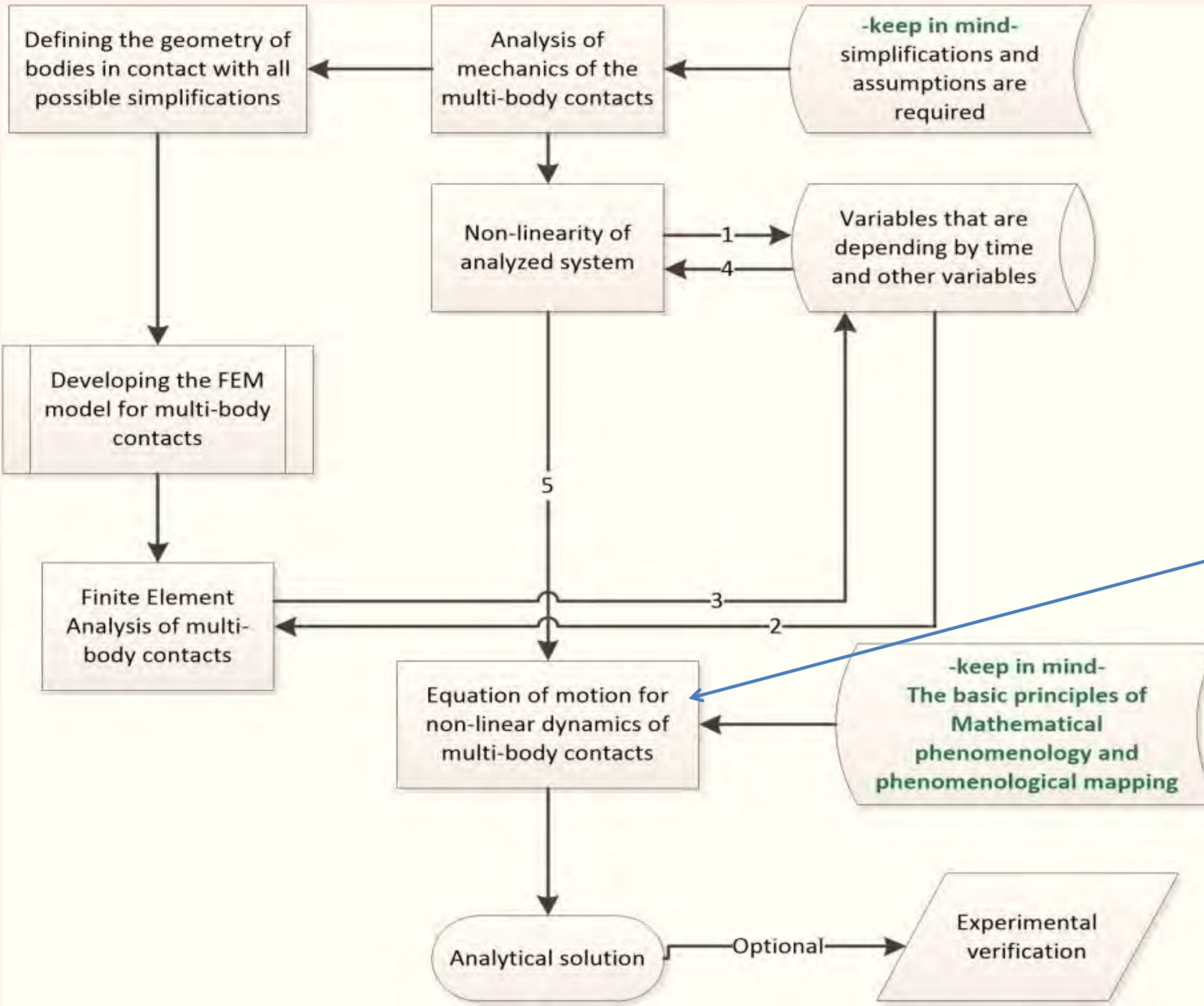
Ivana Atanasovska<sup>1</sup>, Natasa Soldat<sup>2</sup>

<sup>1</sup> Mathematical institute of SASA, Belgrade, Serbia, iviatanasov@yahoo.com

<sup>2</sup> Faculty of Mechanical Engineering, Belgrade, Serbia, nsoldat@mas.bg.ac.rs

## PROCEDURE FOR MULTI-BODY CONTACTS

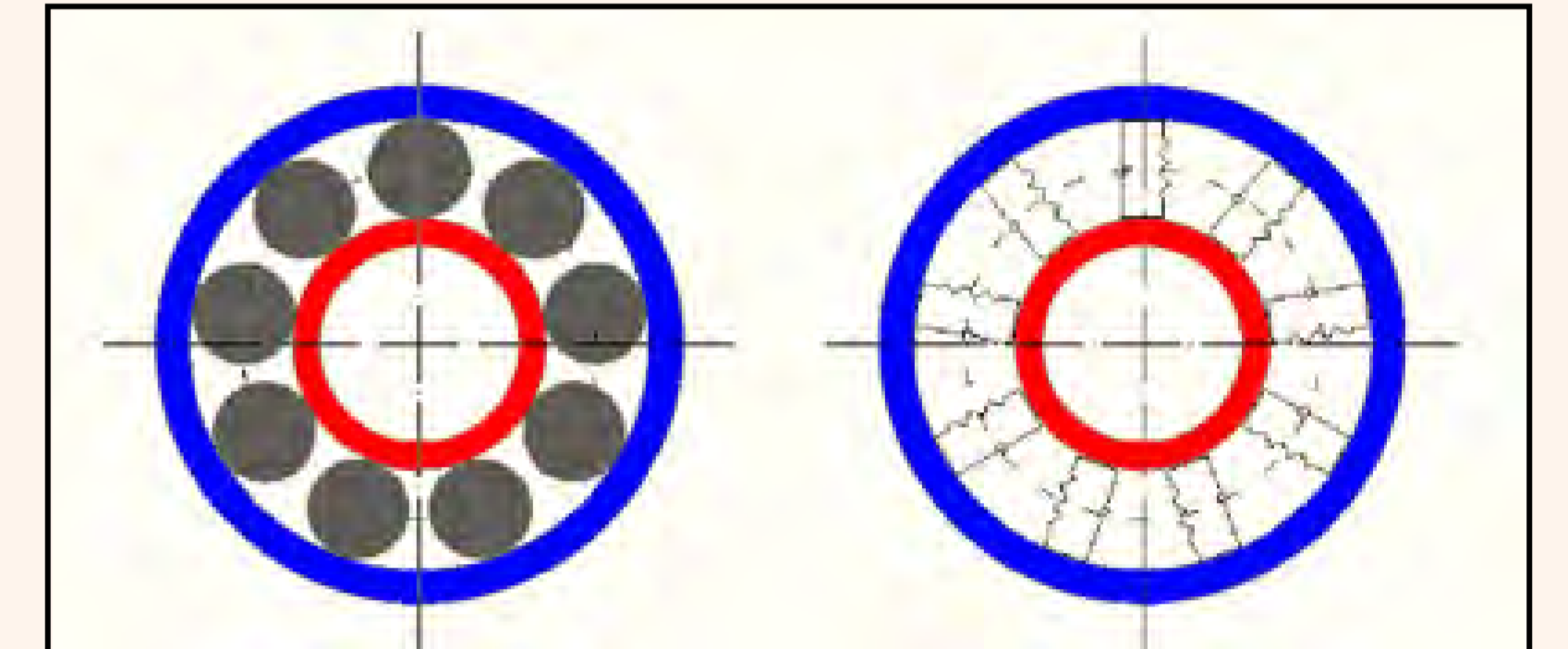
### - FLOW CHART OF THE NEW APPROACH FOR NON-LINEAR DYNAMICS OF MULTI-BODY CONTACTS



The new approach solves the non-linear dynamics of complex mechanical systems, which contains two or more deformable bodies in contact. The developing of presented approach is motivated by problems of solving the vibrations and noise generated by real complex mechanical systems. The analyzed problems is characterized with continuous changes of contact areas' geometry, friction, load distribution and other parameters - Therefore, the problem of non-linear dynamics of mechanical systems with contact pairs becomes very complex.

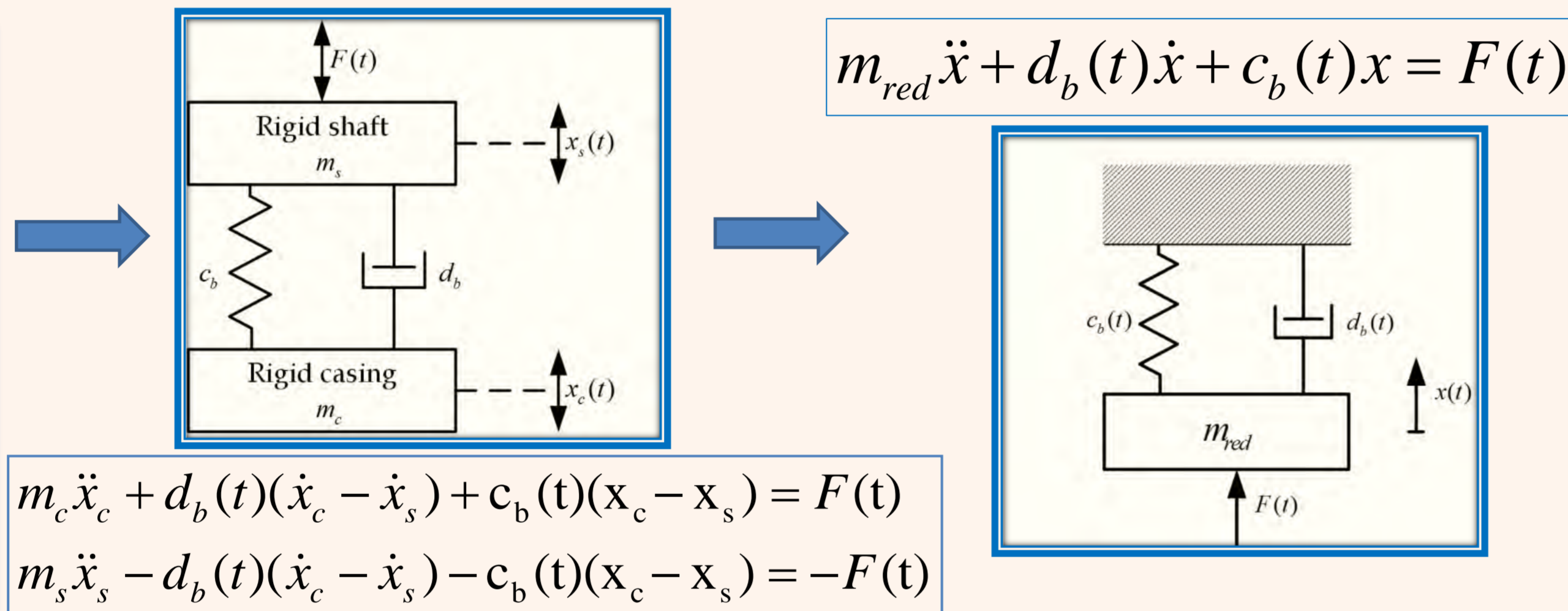
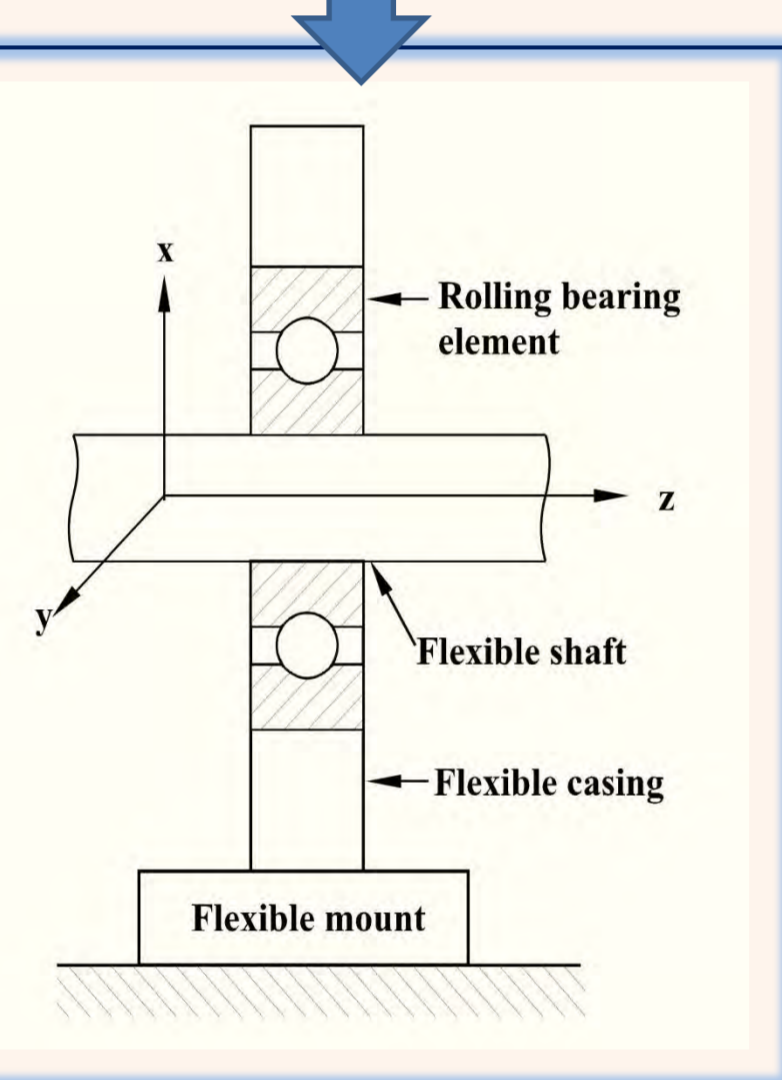
$$[M]\{\ddot{x}(t)\} + [D]\{\dot{x}(t)\} + [C]\{x(t)\} = \{F(t)\}$$

$[M]$ ,  $[D]$  and  $[C]$  – the system mass, damping and stiffness matrices respectively  $\{x(t)\}$  and  $\{F(t)\}$  – the generalized displacement and applied load vectors respectively.

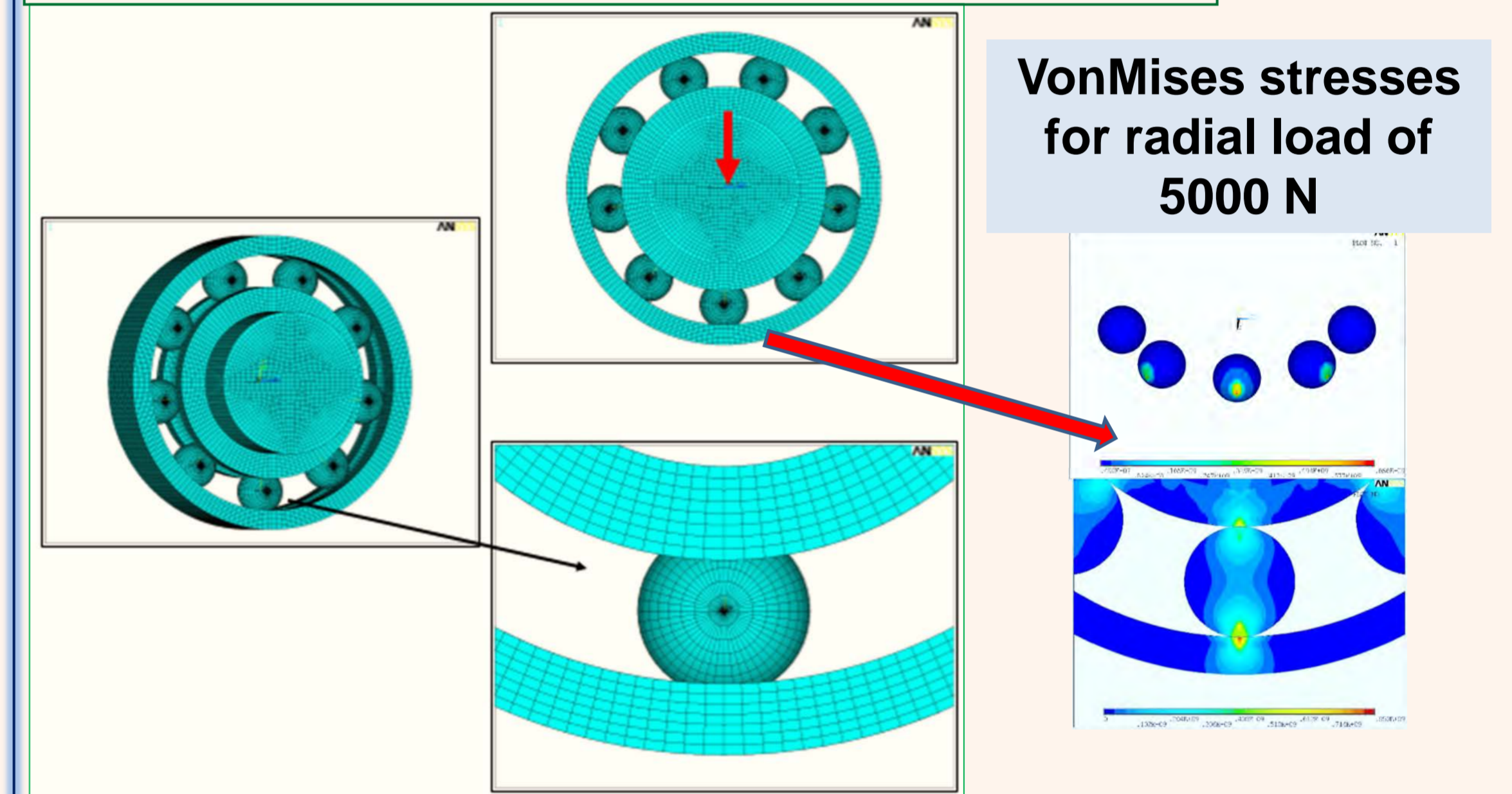


Contact between balls and races are seen as non-linear springs, which operate only in compression, simulating contact deformation and resulting force

## Ball bearing assembly - Simple machine system with rolling ball bearing - reduction to one degree of freedom system for radial ball bearings -



## Contact Finite Element Analysis for ball bearing type 6206 by SKF

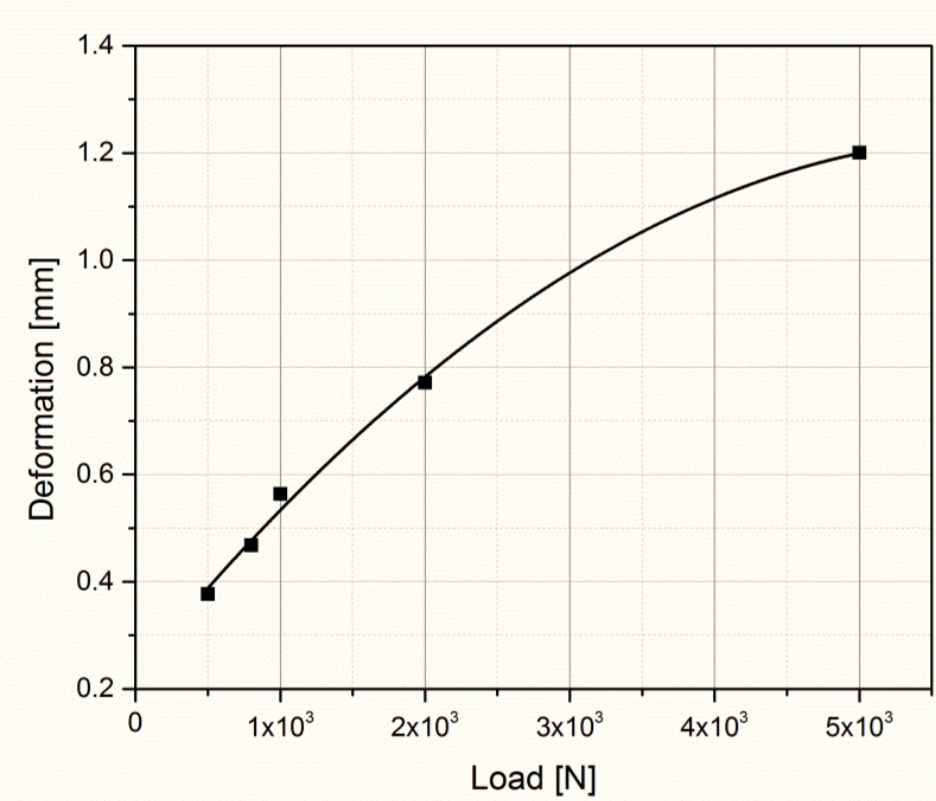


## Finite Element Analysis results

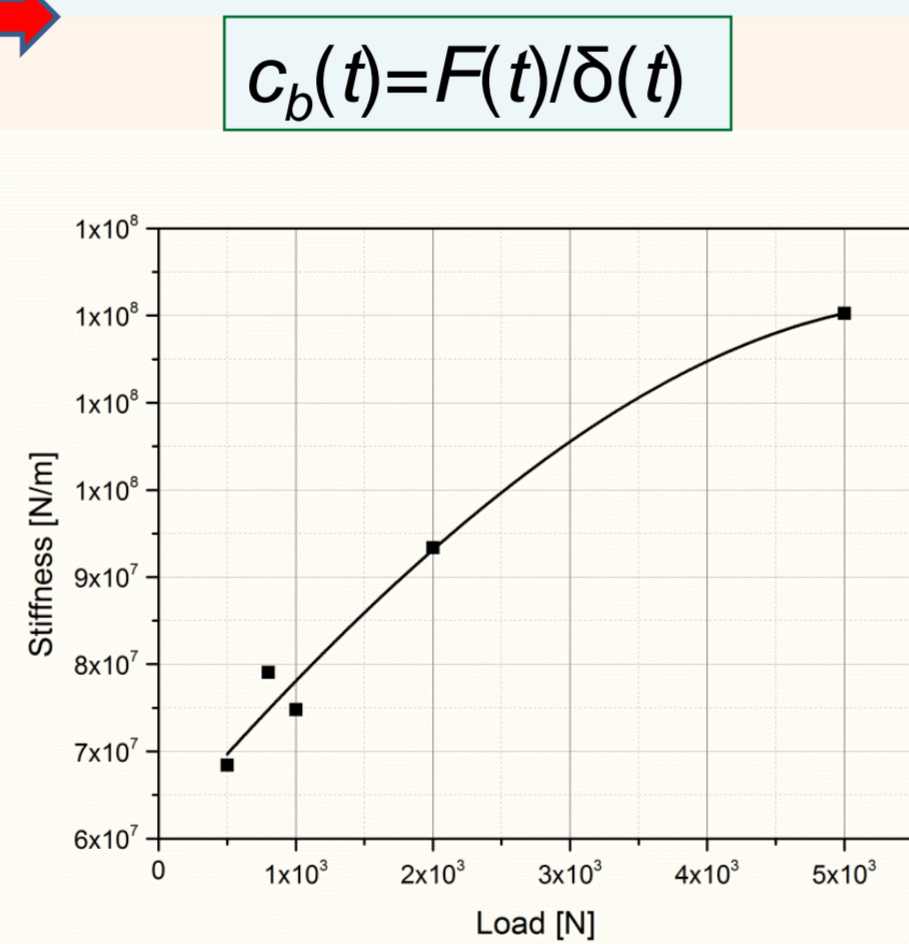
Sample ball bearing – 6206 by SKF

### FEA results for sample without defect

Average values of total deformation of bearing assembly vs external load

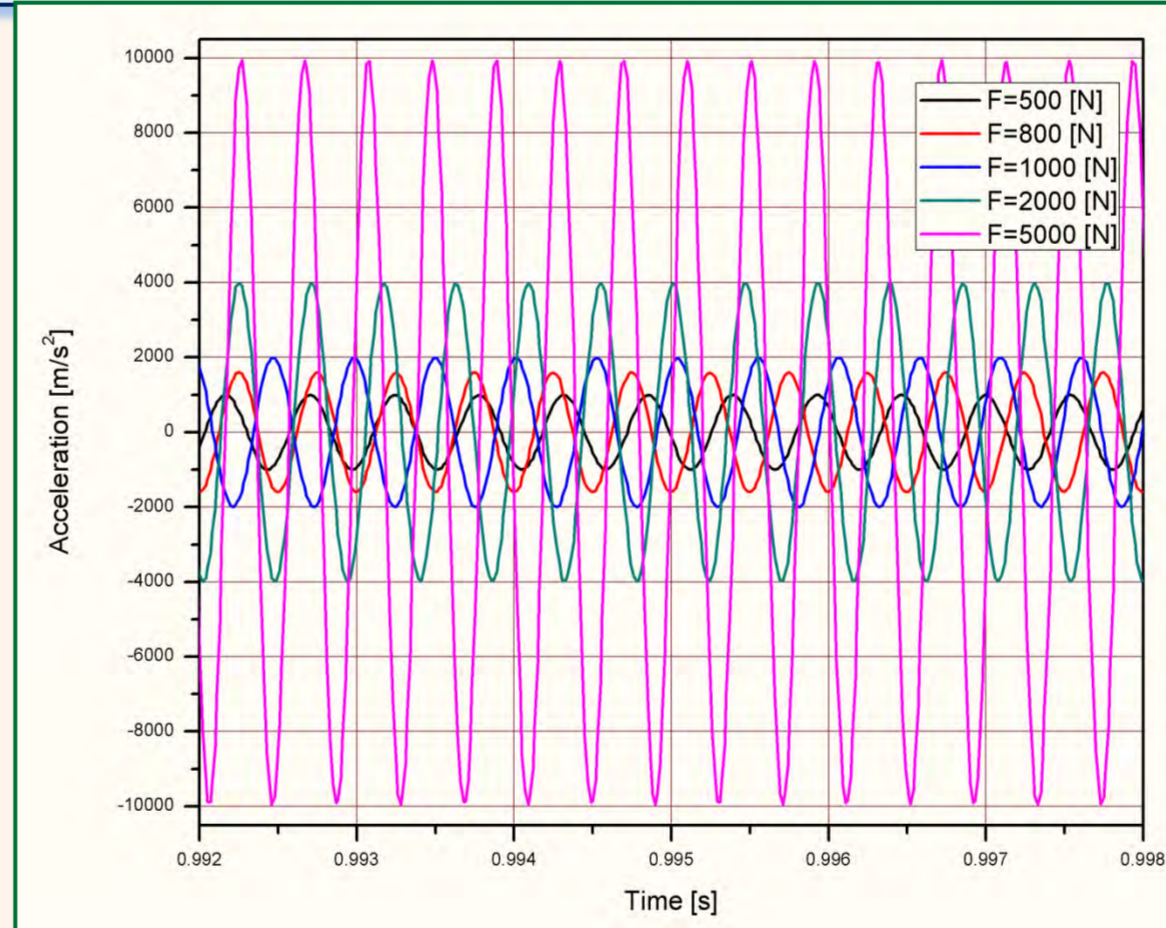


Average radial stiffness vs external load

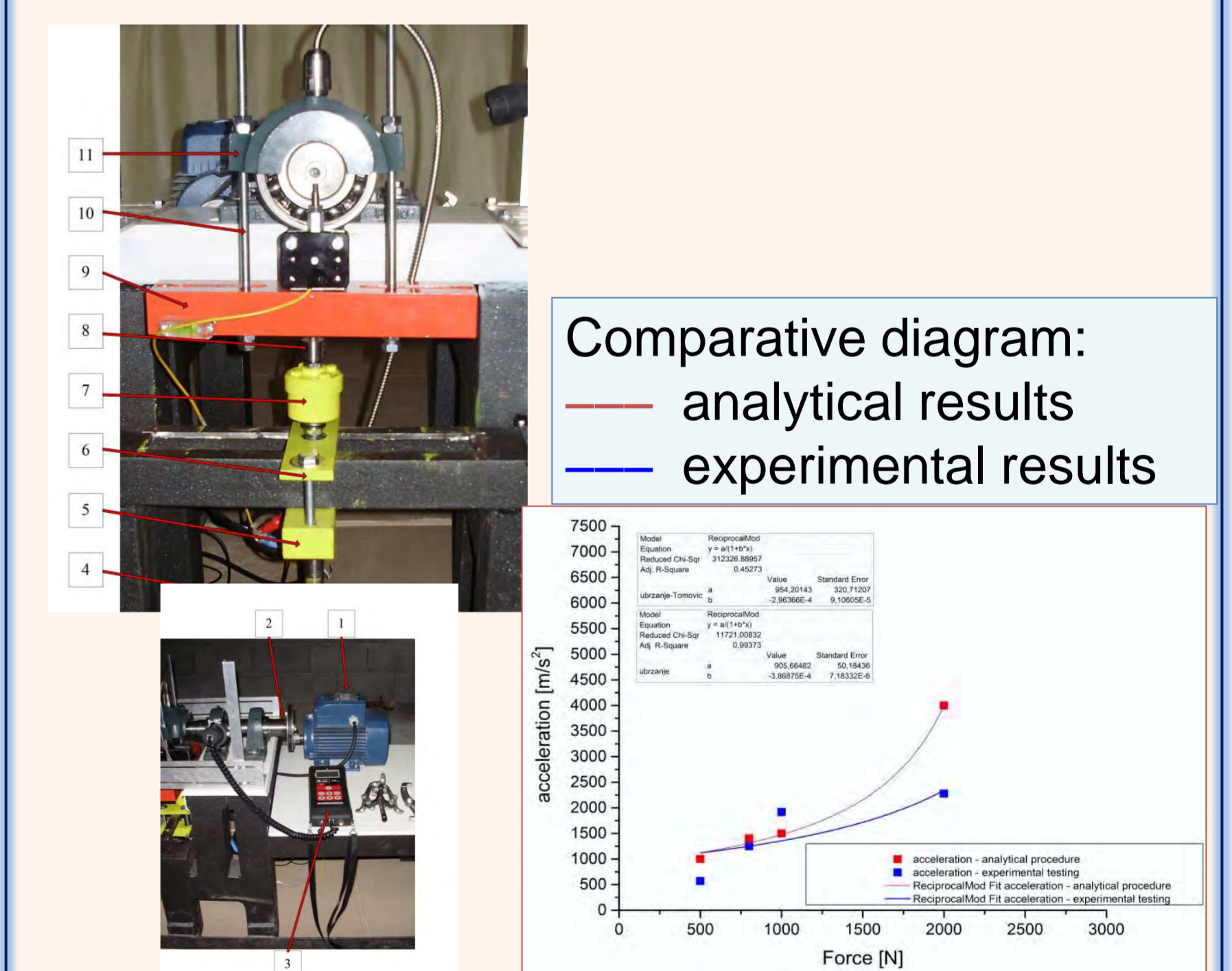


$$m_{red}\ddot{x} + d_b(t)\dot{x} + c_b(t)x = F(t)$$

The Runge-Kutta numerical iterative method

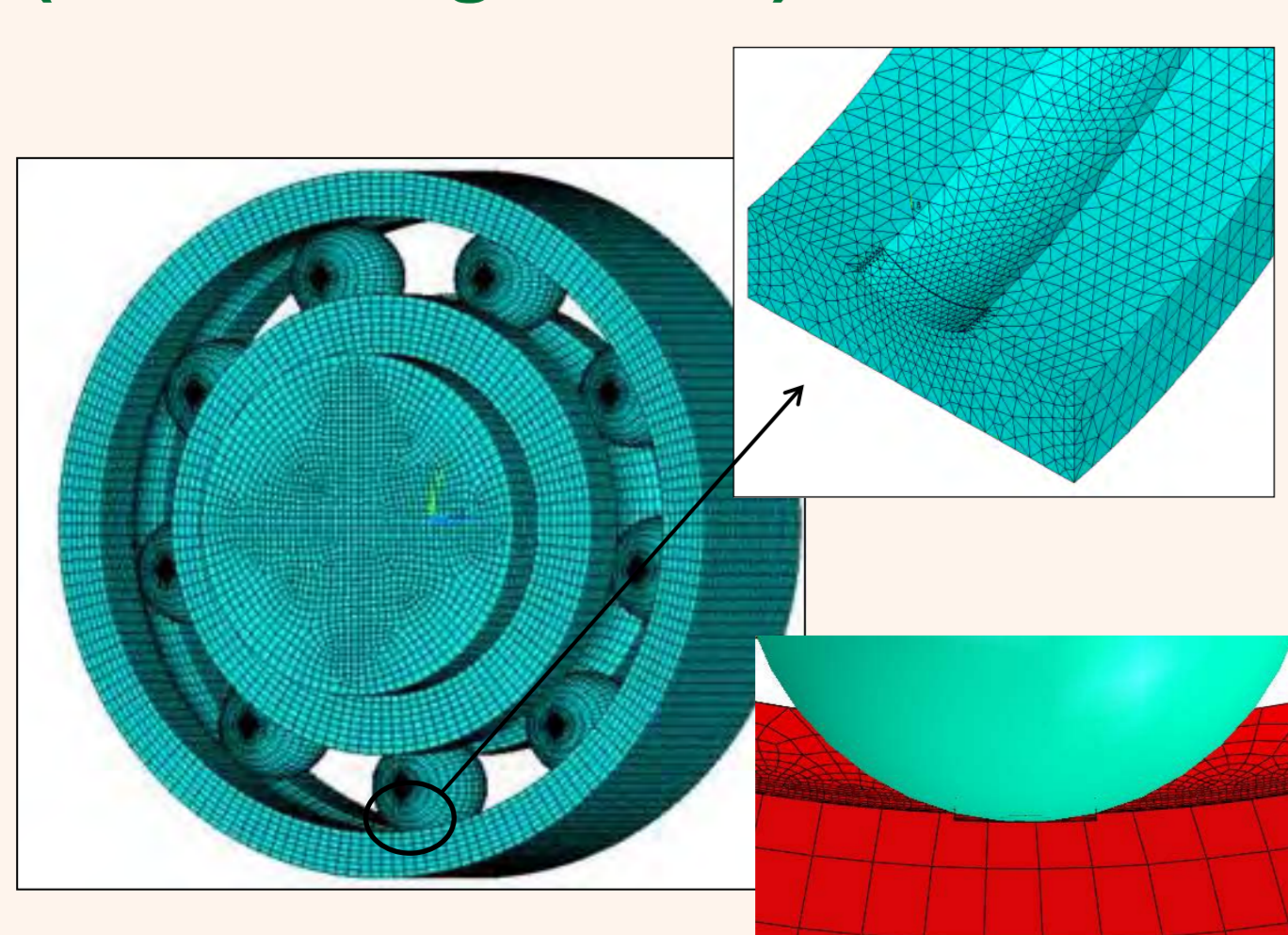


## Verification test ring

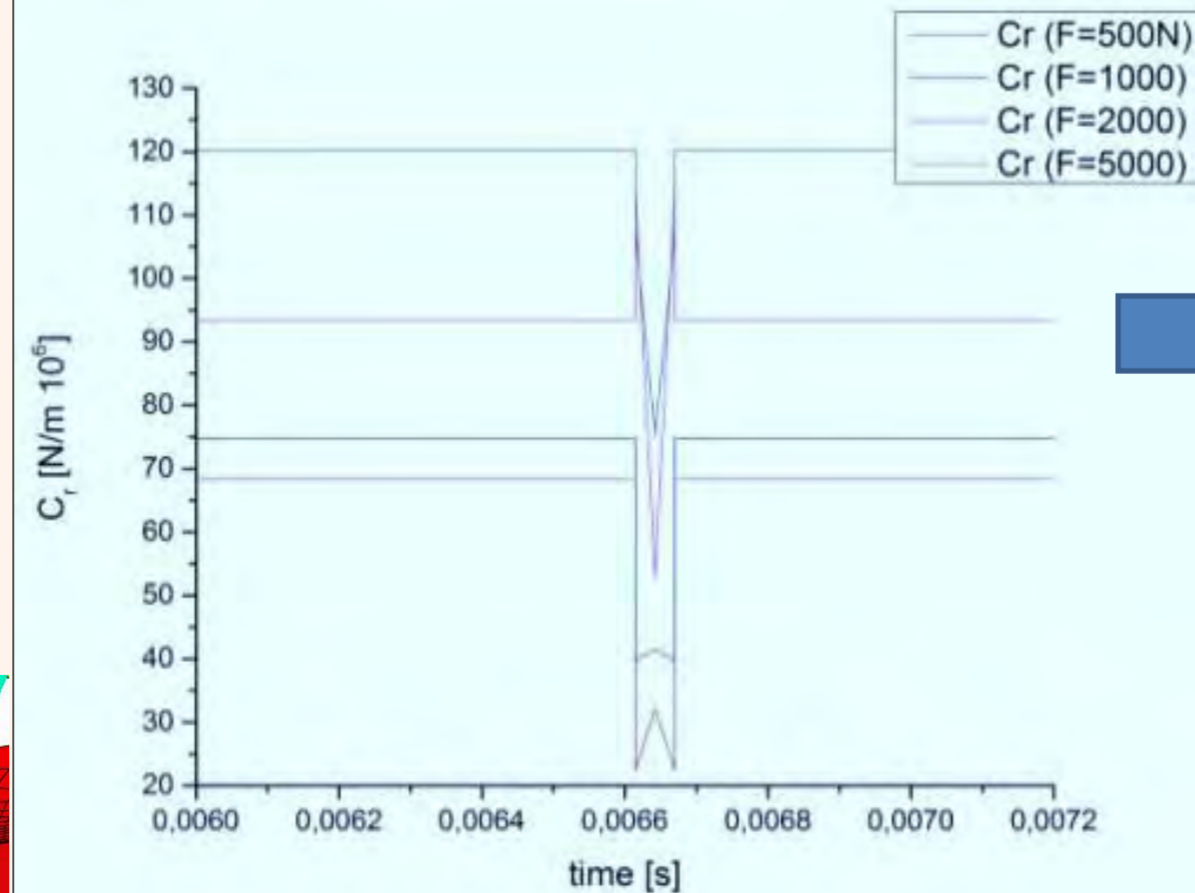


## Ball bearing dynamics when damage exists on raceway on outer ring - Analysis for different external loads

Finite Element Model (with damage detail)



Variable radial stiffness - zoom in detail



Comparative diagrams for ball bearings with and without damage

Velocity-displacement diagrams for different external loads

